Two-Phase Flow Around a Two-Dimensional Cylinder

B. V. R. Vittal* and W. Tabakoff†
University of Cincinnati, Cincinnati, Ohio

The presence of solid particles in many industrial and military applications has a detrimental effect on the performance of the components. This paper presents a numerical solution for the fundamental problem of solid particles suspended in viscous incompressible flow over a cylinder at low flow Reynolds number. The effect of solid particles on flow properties, such as streamline pattern, coefficient of drag of the cylinder, separation angle, and recirculation eddy, are discussed.

Nomenclature

A	= surface area of the particle
a	= acceleration
c_d	= coefficient of drag of the particle
D, D_p	= diameters of the cylinder and particle, respectively
MF_p	= mass fraction of the particles carried by a particular trajectory [Eq. (12)]
m_p	= mass of the particle
r	= radius of the cylinder
Re_d, Re_r	= Reynolds number of the cylinder based on cylinder diameter and radius, respectively
R_x, R_y	= interphase force components
t	= time
U_∞	= freestream velocity
u, v	= velocity components
V	= volume of the element
x, y	= independent Cartesian spatial coordinates
β	= angle between impacting particle velocity and surface
μ	= coefficient of viscosity
ξ, η	= independent spatial coordinates in transformed plane
ρ	= density
τ	= residence time of particle [Eq. (12)]
ψ	= stream function
ω	= vorticity

Subscripts and Superscripts

N	= component normal to the surface
p	= particle
T	= component tangential to the surface
1,2	= before and after impact, respectively
ξ, η	= partial derivatives with regard to ξ and η, respectively
$(\ )'$	= dimensional quantities

Introduction

THE presence of solid particles in many industrial and military applications has a detrimental effect on the performance of the components. The fine metallic powder present in many solid-rocket propellants causes erosion in the rocket nozzle. Further, as the particles lag during high flow accelerations, they may cause deterioration in the performance of the nozzle. In such flows, the particles may have concentrations as high as 40% of the total mass flow and may affect the fluid properties considerably. In the design of industrial filters, it is necessary to predict the particle trajectories in order to assess the efficiency of these filters in capturing solid particles from the carrier fluid. Many gas turbine engines often operate in contaminated environments. Aircraft engines, depending on their mission, may encounter sand, dust, salt, or water droplets. Solid particles are also ingested by gas turbine engines in ground vehicles and by auxiliary power units. Thus, the study of particulated flow is of considerable importance in a number of fields.

The problem of particulated flow in any flow system can be divided into three parts. The first part consists of the particle trajectories. The second is the effect of the presence of the particles on the flow behavior. The third part consists of the nature of the solid surface impact and the material erosion. Considerable research has been done on multiphase flowfields for inviscid carrier fluid. These studies have concentrated on the calculation of particle trajectories in various flow systems.[1,2] There are a number of published reports available on the prediction of erosion of materials due to particle impacts. There is not much information available, however, concerning the effect of the presence of solid particles on the carrier fluid properties. This effect is investigated in this paper for solid-particle flow in a viscous fluid.

The study of plane uniform viscous flow around a cylinder is a fundamental problem because all difficulties that arise in this flowfield are amplified for obstacles of other shapes. This study of flow around a cylinder is basic to the calculation of more complex flows, such as flow over airfoils. Most of the work done so far on two-phase (solid particle/gas) flow over a cylinder is based on the inviscid approach.[3,4] The path of the particles changes appreciably when the viscosity of the continuous phase is taken into account. The present analysis of viscous fluid/solid-particle flow over a two-dimensional cylinder takes into account the effect of the presence of the particles everywhere, including those rebounded off the cylinder surface.

Governing Equations

In the case of solid-particle/gas flow, where the volume fraction of the particles is low, the Lagrangian formulation of the equations of motion is most appropriate for the particulate phase, while the Eulerian approach is convenient for the continuous phase. Further, if the Eulerian approach is to be used for the particle phase, it becomes extremely difficult to define the boundary conditions at the solid boundaries, where the particles deflect after impact.

Presented as Paper 86-0349 at the AIAA 24th Aerospace Sciences Meeting, Reno, NV, Jan. 6–9, 1986; received Jan. 30, 1986; revision received Sept. 1, 1986.

*Department of Aerospace Engineering and Engineering Mechanics; presently working at Allison Gas Turbine Division, General Motors, Indianapolis, IN. Member AIAA.

†Professor, Department of Aerospace Engineering and Engineering Mechanics. Fellow AIAA.

Eulerian Formulation for the Fluid

For two-dimensional incompressible flow, if the volume fraction of the solid particles is low, the governing equations in Cartesian coordinates are[5,6]

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\rho'\left(\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'}\right) = -R'_x - \frac{\partial p'}{\partial x'} + \mu\left(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2}\right) \tag{2}$$

$$\rho'\left(\frac{\partial v'}{\partial t'} + u'\frac{\partial v'}{\partial x'} + v'\frac{\partial v'}{\partial y'}\right) = -R'_y - \frac{\partial p'}{\partial y'} + \mu\left(\frac{\partial^2 v'}{\partial x'^2} + \frac{\partial^2 v'}{\partial y'^2}\right) \tag{3}$$

where R'_x and R'_y are interphase force terms depicting the presence of the solid particles. Eliminating the pressure terms from the above equations and introducing vorticity, $\omega' = \partial v'/\partial x' - \partial u'/\partial y'$, one obtains

$$\rho'\left(\frac{\partial \omega'}{\partial t'} + u'\frac{\partial \omega'}{\partial x'} + v'\frac{\partial \omega'}{\partial y'}\right) = -\frac{\partial R'_y}{\partial x'} + \frac{\partial R'_x}{\partial y'} + \mu\left(\frac{\partial^2 \omega'}{\partial x'^2} + \frac{\partial^2 \omega'}{\partial y'^2}\right) \tag{4}$$

Substituting for u' and v' in terms of stream functions ψ and nondimensionalizing the various quantities with respect to freestream velocity U_∞ and radius r of the cylinder, one obtains the following equation:

$$\frac{\partial \omega}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \omega}{\partial y} = -\frac{\partial R_y}{\partial x} + \frac{\partial R_x}{\partial y} + \frac{1}{Re_r}\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) \tag{5}$$

For general use, it is convenient to express these equations in a coordinate system in which ξ and η are two independent spatial variables. Equation (5) becomes

$$J\frac{\partial \omega}{\partial t} + \frac{\partial \psi}{\partial \eta}\frac{\partial \omega}{\partial \xi} - \frac{\partial \psi}{\partial \xi}\frac{\partial \omega}{\partial \eta} = y_\xi\frac{\partial R_y}{\partial \eta} - y_\eta\frac{\partial R_y}{\partial \xi} + x_\xi\frac{\partial R_x}{\partial \eta} - x_\eta\frac{\partial R_x}{\partial \xi} + \frac{1}{Re_r}\left[\frac{\partial}{\partial \xi}\left(\frac{\alpha}{J}\frac{\partial \omega}{\partial \xi} - \frac{\beta}{J}\frac{\partial \omega}{\partial \eta}\right) + \frac{\partial}{\partial \eta}\left(\frac{\gamma}{J}\frac{\partial \omega}{\partial \eta} - \frac{\beta}{J}\frac{\partial \omega}{\partial \xi}\right)\right] \tag{6}$$

where

$$\alpha = x_\eta^2 + y_\eta^2, \qquad \beta = x_\xi x_\eta + y_\xi y_\eta$$
$$\gamma = x_\xi^2 + y_\xi^2, \qquad J = x_\xi y_\eta + x_\eta y_\xi \tag{7}$$

The relationship between vorticity ω and the stream function ψ in the general coordinate system is given by

$$-J\omega = \frac{\partial}{\partial \xi}\left(\frac{\alpha}{J}\frac{\partial \psi}{\partial \xi} - \frac{\beta}{J}\frac{\partial \psi}{\partial \eta}\right) + \frac{\partial}{\partial \eta}\left(\frac{\gamma}{J}\frac{\partial \psi}{\partial \eta} - \frac{\beta}{J}\frac{\partial \psi}{\partial \xi}\right) \tag{8}$$

Lagrangian Formulation for the Particles

The trajectory of a particle in a moving fluid is governed by the vector balance of its rate of change of momentum and the external forces acting on it. Many external forces may act on the particles. For large ratios of particle material density to gas density, the force acting on a spherical particle due to the flow pressure gradient can be neglected when compared to the drag force on it. If the volume fraction of the particles is low, particle/particle interaction is neglected.

An excellent review of the forces acting on the particles and their relative importance in the fluid flow calculations is given in Ref. 7. The drag of the particle, which is the predominant force, is usually calculated based on empirical relations.

The equations of motion of a single particle in Cartesian coordinates are given by

$$m'_p\frac{du'_p}{dt'} = \frac{\pi}{8}C_d\rho'(D'_p)^2(u'_p - u')\left[(u' - u'_p)^2 + (v' - v'_p)^2\right]^{\frac{1}{2}} \tag{9}$$

$$m'_p\frac{dv'_p}{dt'} = \frac{\pi}{8}C_d\rho'(D'_p)^2(v'_p - v')\left[(u' - u'_p)^2 + (v' - v'_p)^2\right]^{\frac{1}{2}} \tag{10}$$

Impact and Rebound Phenomenon of Particles

When solid particles move in a stream of gas, they do not generally follow the streamlines taken by the gas because of the higher inertia (of the solid particles). They collide with the solid boundaries, after which the particles experience a loss in their momentum relative to the wall and change the direction of their motion. The value of the velocity of a particle and the direction of its motion as it rebounds from the surface after collision must be known so that the solution of the particle equations of motion may be continued beyond the points of collision. Particle rebound characteristics have been studied experimentally by Tabakoff et al.[8] The particle rebound characteristics are statistical in nature. It has been found, however, that they are a strong function of impingement angle. Although there is considerable scatter in the rebound characteristics, Tabakoff et al.[8] were able to represent them with mean restitution ratios as a function of impingement angle. The following empirical relations for the mean rebound-to-impact restitution ratios are used in trajectory calculations:

$$V'_{p_{2_T}}/V'_{p_{1_T}} = 1.0 - 2.12\beta'_p + 3.0775\beta'^2_p - 1.1\beta'^3_p$$
$$V'_{p_{2_N}}/V'_{p_{1_N}} = 1.0 - 0.4159\beta'_p + 0.4994\beta'^2_p - 0.292\beta'^3_p \tag{11}$$

where V'_{p_N} and V'_{p_T} represent the particle velocity components normal and tangent to the solid surface and the subscripts 1 and 2 refer to the conditions before and after impact, respectively. In the above equations, β'_p is the angle between the impact velocity and the tangent to the surface in radians.

Numerical Procedure

The stream function and vorticity transport form of the equations of motion of fluid, including the interphase force terms, are solved using a factored alternating direction implicit (ADI) scheme developed by Davis et al.[9,10] Symmetry is assumed along the stagnation streamline; therefore, the solution is presented only for the upper half-plane. The flow in the upper half is bounded by the half-cylinder of one radius; the outer flow boundary is at a distance 49 times the radius. This region is transformed to an ξ, η solution plane in such a way that ξ is a coordinate tangent to the body surface and η is a coordinate normal to the body surface. The equations are solved employing a (65×65) grid for the solution plane. Symmetry boundary conditions of $\psi = 0$ and $\omega = 0$ are used

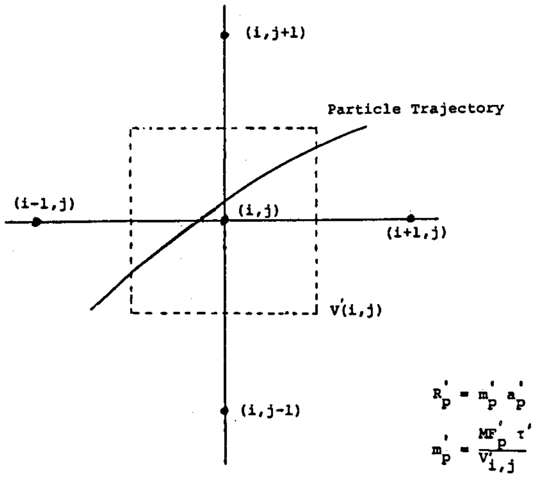

Fig. 1 Grid system.

along symmetry boundaries. For the outer flow, the vorticity boundary condition $\omega = 0$ is used. The stretching in the physical plane is accomplished by using appropriate stretching parameters. The boundary condition prescribed on the symmetry axis does not permit unsteady oscillatory wake to occur. So the analysis is limited to low Reynolds number flows.

The interphase force terms in the fluid flow equations are evaluated separately using particle trajectory calculations as employed by Crowe.[11] Particles are introduced upstream of the cylinder at various heights. Uniform entry of the particles with the flow is approximated by 250 discrete entry locations, each location is assumed to carry a fraction of the total particle mass. First the flowfield is established by solution without solid particles. Using this flowfield, the particle flow path and its properties along the path are calculated. Referring to Fig. 1, the force per unit volume R_p' at the grid point (i, j) is given by

$$R_p' = m_p' a_p' \tag{12}$$

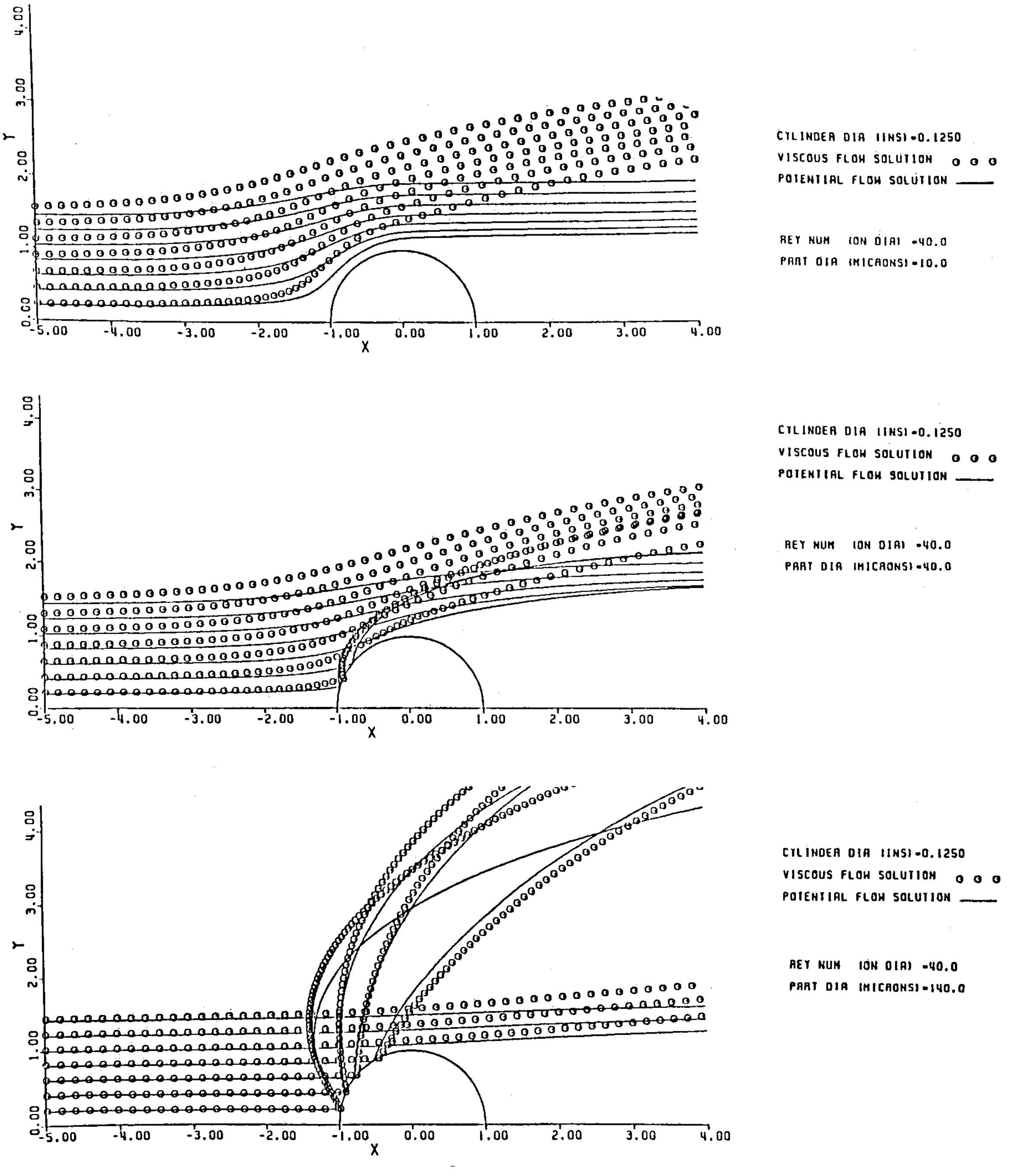

Fig. 2 Particle flow path over cylinder.

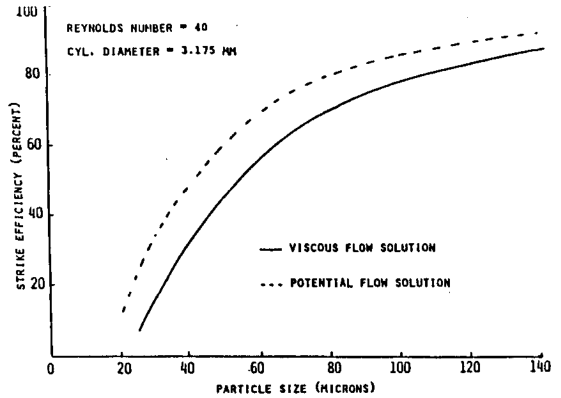

Fig. 3 Effect of particle size on strike efficiency.

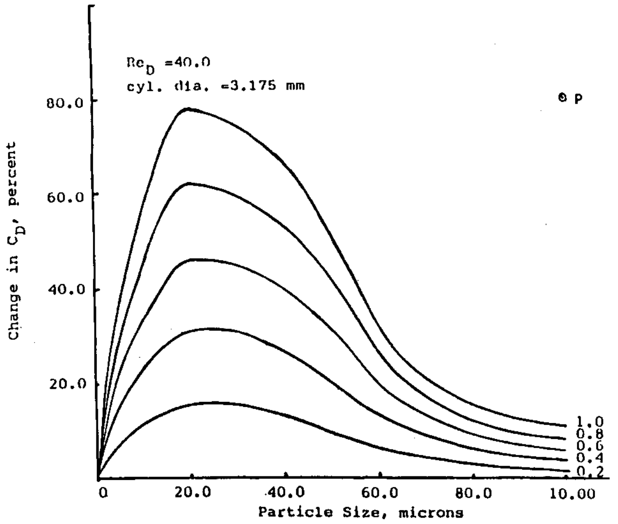

Fig. 4 Effect of particle size on cylinder C_D.

where

$$m'_p = MF_p \tau / V_{i,j}$$

MF_p is the mass fraction of the particles carried by the particular trajectory, and τ is its residence time in volume, $V_{i,j}$.

The total mass fraction of the particles is defined as the ratio of the particle mass flow to that of the gas flow. Equation (12) is integrated along the particle trajectory for each time step and for each grid point. The total force R'_p at the grid point (i, j) is the sum of all the forces due to all the particle trajectories passing through the mesh control volume.

$$R'_{p(i,j)} = \sum_{k=1}^{N} (m'_p a'_p)_k \tag{13}$$

Results and Discussion

Numerical solutions are obtained for two-dimensional incompressible viscous particulate flow over a cylinder. Air at atmospheric pressure and temperature is considered for the continuous phase, and quartz solid particles are used for the particulate phase. The flow calculations are made for a cylinder diameter of 3.175 mm. At low cylinder Reynolds numbers, the coefficient of drag of the cylinder changes appreciably for slight changes in flow conditions. So any change in the flow properties due to the presence of the particles would be reflected on the fluid streamline pattern and the coefficient of drag of the cylinder. The following operating conditions are used:

Freestream pressure = 10,366 kg/m^2
Freestream temperature = 288 K
Air density = 0.01273 kg/m^3
Cylinder diameter = 3.175 mm
Particle material = quartz
Particle material density = 2444 kg/m^3
Kinematic viscosity of air = 1.46×10^{-5} m^2/s
Cylinder Reynolds number = $U_\infty D/\nu$; where D is the diameter of the cylinder

Comparison of Particle Trajectories in Viscous and Inviscid Fluid

The particle trajectory is affected by fluid flow path and its properties. Figure 2 shows the particle trajectories for different particle sizes. Besides the obvious difference in particle trajectories among particles of different sizes, there is considerable difference in the particle trajectories between viscous and inviscid solutions. It can be observed that smaller particles are deviated away from the cylinder and do not impact on the cylinder surface. Larger particles, because of their inertia, tend to keep their initial momentum, impact on the surface of the cylinder, and are deflected. In the case of viscous flow solutions, the boundary layer creates a displacement thickness altering the apparent size of the cylinder, thus changing the particle path when viscosity is taken into account.

In Fig. 3, the collection efficiency or the strike efficiency is given for various particle sizes for both viscous and inviscid flow cases. This efficiency, a number frequently quoted in industrial literature, is the ratio of the number of particles impacting with the object to the number of particles that would impact if they followed straight-line trajectories without deflection by the gas. Clearly, the effect of viscosity is to reduce the strike efficiency for a given particle size. Most of the particles less than 20 μm in diameter do not strike the cylinder at all.

Effect of Particles on Fluid Flow Properties

The presence of particles in the fluid could affect such flow properties as streamline pattern, vorticity, coefficient of drag, separation angle, and recirculation eddy. The overall effect of the particles would be readily reflected on the coefficient of drag of the cylinder. Figure 4 shows the percentage change in coefficient of drag of the cylinder plotted against particle size for different particle mass fractions. The drag represented is the result of the aerodynamic forces on the cylinder and does not include the force due to impact of the particles. It is interesting to observe that initially, as the particle size increases, the coefficient of drag increases. With the increase in size of the particle, there is a bigger difference in relative velocities between the particle and the air because of higher inertia, thus resulting in higher interphase forces. This changes the fluid velocities, resulting in increased coefficient of drag of the cylinder. However, with the increase in particle size, for the same mass fraction of the particles, fewer particles are present in the flow. This decreases the interphase forces, thereby affecting the coefficient of drag of the cylinder. Thus, the influence of the particles on the fluid flow depends on the size and number of particles present for a given mass fraction.

The presence of the particles is felt on all the fluid flow parameters. Figure 5 shows the effect of particles on the pressure distribution over the cylinder surface. The pressure coefficient C_p is presented at various angular positions on the cylinder.

The effect of particles on fluid flow can easily be seen in the way it changes the fluid streamline pattern. Figure 6 shows the effect of particle concentration on fluid streamline pattern for 10-μm-diam particles. There is a change in the recirculation pattern of the fluid downstream of the cylinder. With increase in particle mass fraction, the size of the recirculation zone increases. It may be recalled that smaller diameter particles in the 10-μm range do not impact on the cylinder. As the diameter of the particle increases, more particles impact on

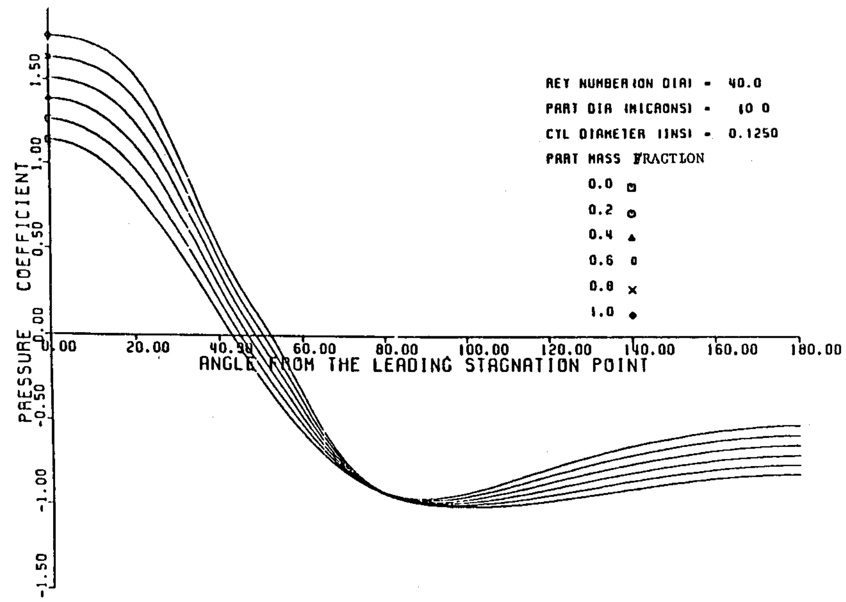

Fig. 5 Pressure coefficient variation with particle mass fraction.

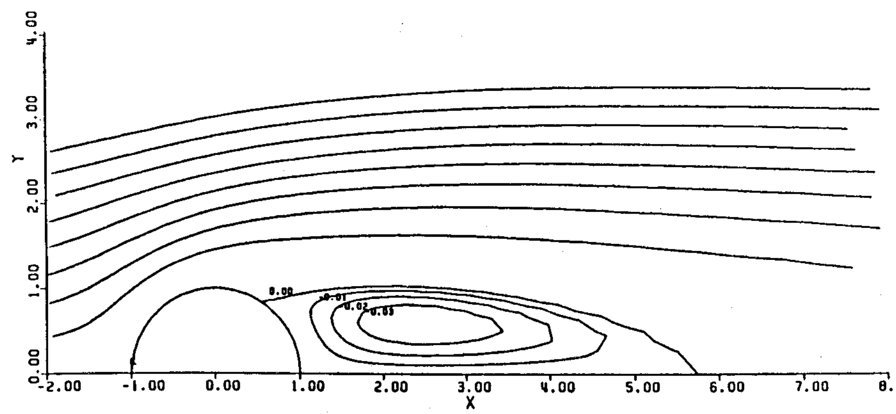

Fig. 6a Streamlines without particles.

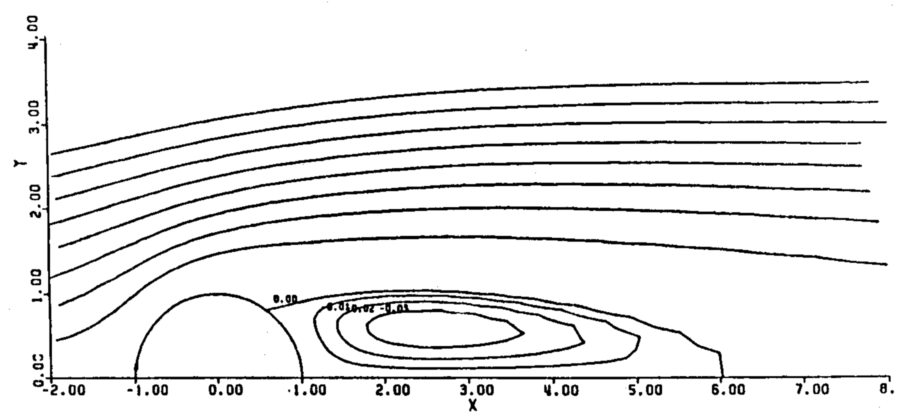

Fig. 6b Streamlines for 0.20 mass fraction.

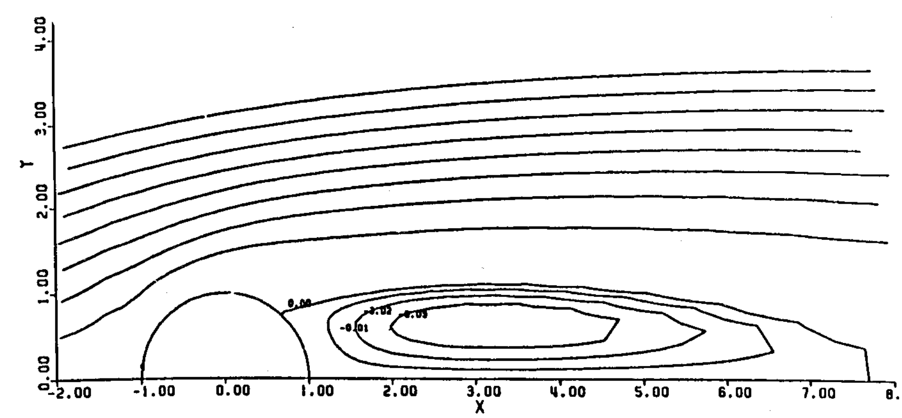

Fig. 6c Streamlines for 0.60 mass fraction.

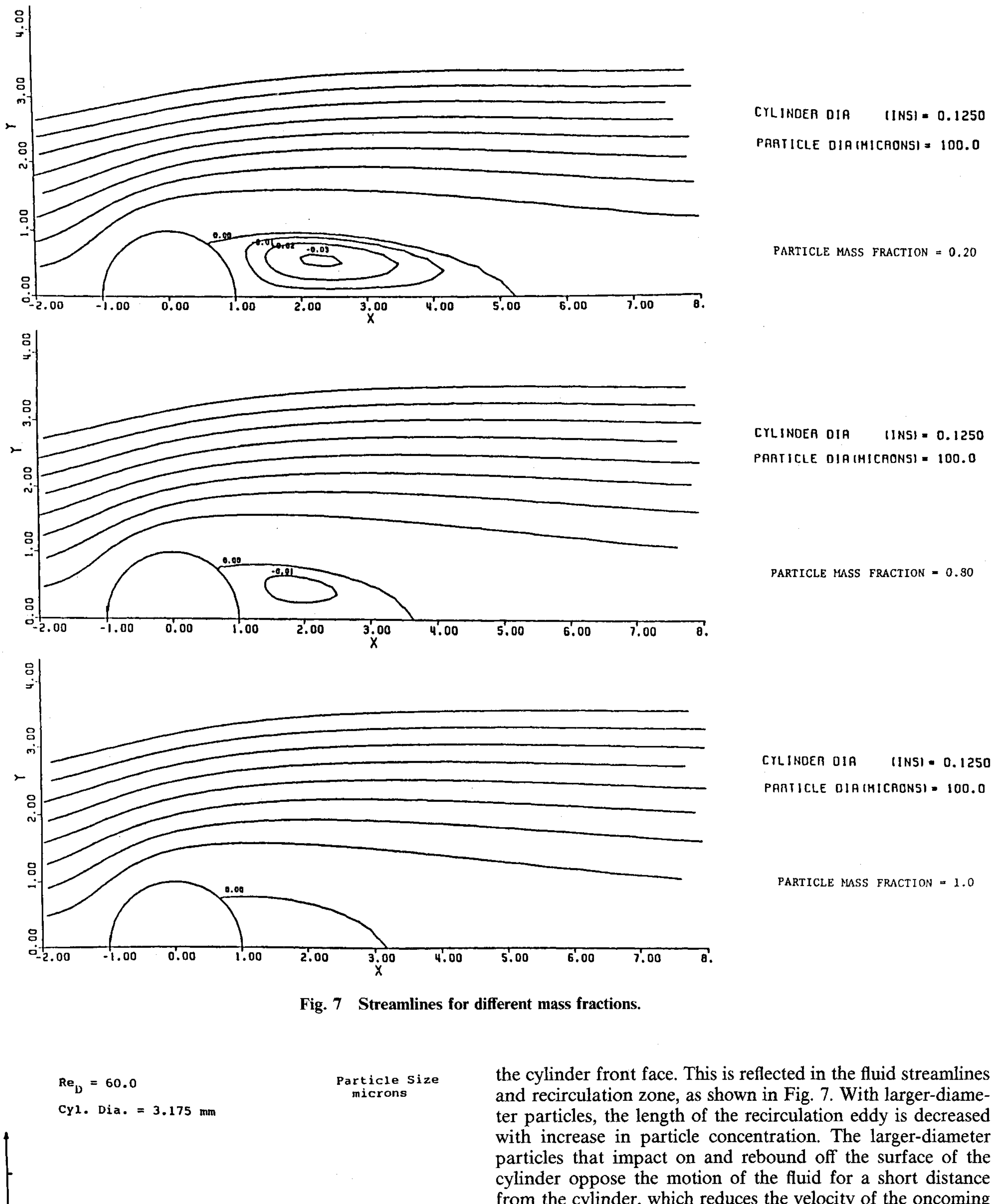

Fig. 7 Streamlines for different mass fractions.

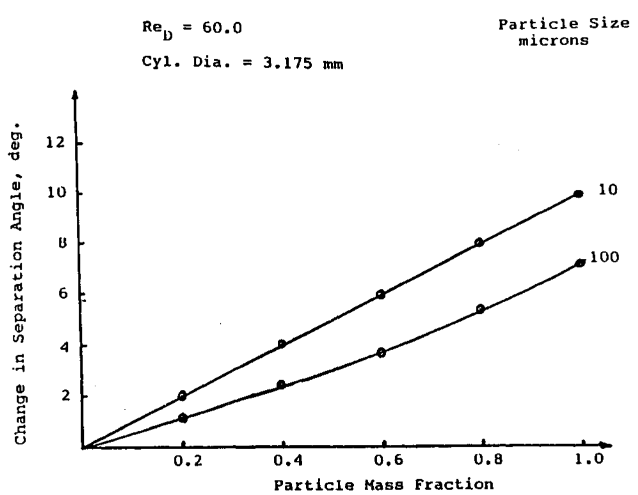

Fig. 8 Effect of particle mass fraction on separation angle.

the cylinder front face. This is reflected in the fluid streamlines and recirculation zone, as shown in Fig. 7. With larger-diameter particles, the length of the recirculation eddy is decreased with increase in particle concentration. The larger-diameter particles that impact on and rebound off the surface of the cylinder oppose the motion of the fluid for a short distance from the cylinder, which reduces the velocity of the oncoming fluid. When fluid loses momentum to drive the particles, it is as if the cylinder is being run at lower freestream velocity and thus at lower Reynolds number.

The change in flow separation angle with particle concentration is shown in Fig. 8. Once again, it can be observed that the particles have a substantial effect on the separation angle. All these changes in the flow properties are reflected on the coefficient of drag of the cylinder.

Conclusion

The solid-particle trajectories over a two-dimensional cylinder are shown to be different for viscous and inviscid fluid and for different particle sizes. The particle strike or collection efficiency is found to be different for viscous and inviscid solutions. It has been shown that the presence of the particles has a substantial effect on the fluid flow properties. The particles change important flow parameters such as coefficient of drag, separation zone, and recirculation pattern.

Acknowledgments

This work was sponsored by the U.S. Army Research Office—Durham, under Contract DAAG29-K-0029.

References

[1]Beacher, B. and Tabakoff, W., "Improved Particle Trajectory Calculations through Turbomachinery Affected by Coal Ash Particles," ASME Transactions, *Journal of Engineering for Power*, Vol. 104, Jan. 1982, pp. 64–68.

[2]Tabakoff, W. and Hussein, M.F., "Effect of Suspended Solid Particles on the Properties in Cascade Flow," *AIAA Journal*, Vol. 9, Aug. 1971, pp. 1514–1519.

[3]Morsi, S.A. and Alexander, A.J., "An Investigation of Particle Trajectories in Two Phase Flow System," *Journal of Fluid Mechanics*, Vol. 55, 1972, pp. 193–208.

[4]Val Healy, J., "Perturbed Two Phase Cylindrical Type Flows," *The Physics of Fluids*, Vol. 13, March 1970, pp. 551–560.

[5]Marble, F.E., "Dynamics of Dusty Gases," *Annual Review of Fluid Mechanics*, Vol. 2, 1970, pp. 369–377.

[6]Soo, S.L., "Fluid Dynamics of Multiphase Systems," Ginn, Lexington, MA, 1967.

[7]Hamed, A., "Analysis of Nonequilibrium Flow of Viscous Fluid With Suspended Solid Particles," Ph.D. Dissertation, University of Cincinnati, OH, 1972.

[8]Tabakoff, W. and Hamed, A., "Aerodynamic Effects on Erosion in Turbomachinery," Joint Gas Turbine Conference, Tokyo, Japan, JSME/ASME Paper 70, May 1977.

[9]Davis, R.T., "Numerical Solution of the Navier-Stokes Equations for Symmetric Laminar Incompressible Flow Past a Parabola," *Journal of Fluid Mechanics*, Vol. 51, 1972, pp. 417–433.

[10]Hill, J.A., Davis, R.T., and Slater, G.L., "Development of a Factored ADI Scheme for Solving the Navier-Stokes Equations in Stream Function-Vorticity Variables," University of Cincinnati, OH, Rept. AFL 79-12-49, 1979.

[11]Crowe, C.T. and Stock, D.E., "A Computer Solution for Two Dimensional Fluid-Particle Flows," *International Journal for Numerical Methods in Engineering*, Vol. 10, 1976, pp. 185–196.